Cooperation and communication dynamics

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PSE

What do we model?

Dynamic interactions

- An agent's decisions at some point in the game influence other agent's behaviors in the future.
- This part of the course is mostly concerned with long-term interactions.
- Drawn on the theory of repeated games.

Natural applications

- Interaction between firms,
- consumer and producer,
- repeated contracting,
- social norms, customs etc.

What are we after and how?

Cooperation

- Under what circumstances can repetition lead to cooperation?
- More generally, how does repetition affect sustainable outcomes?
- What social norms do we expect in repeated interactions?

Information dynamics

- What is the best way to use private information?
  - Using information reveals it, making it less valuable in the future.
  - Should one use information or conceal it?
  - Is all private information revealed in the long-run?
- Is it possible to create anticipations for a certain behavior?
  - Can a central bank, creditor, producer, or chef build a reputation?
  - How does reputation building influence interactions?
  - Are reputations always good?
  - Do reputations survive in the long-run?

Methodology

Rules of the game

- What do the players initially know?
- What do they observe as the game is played?
- Define strategies in the repeated interaction
- Define preferences (payoffs) in the repeated interaction

Solving and interpreting

- Use equilibrium concepts: Nash equilibria and refinements,
- Draw predictions on player's behavior and on equilibrium payoffs
- Interpret. Are we satisfied with the predictions of the theory?
- If not, is the model or solution concept not appropriate?

Roadmap

A first example

	H	U
G	$1,1$	$-1,-1$
S	$2,0$	$-1,-1$

- What are the Nash equilibria of this game?
- Consider the game played twice. After the first interaction, actions in the first interaction are observed.
  - Draw the game in extensive form
  - What is a strategy for player 1? For player 2? How many of them are there?
  - Do (or don't) represent the game in normal form
  - What are the Nash equilibria?
  - How does the presence of a dominated strategy affect the Nash equilibria of the repeated game?
  - What are the subgames?
  - What are the SPNE of the repeated game?

A second example

	H	U	B
G	$1, 1$	$-1, -1$	$-5, -5$
S	$2, 0$	$-1, -1$	$-5, -5$
B	$-5, -5$	$-5, -5$	$-4, -4$

- What are the Nash equilibria of this game?
- Consider the game played twice. After the first interaction, actions in the first interaction are observed.
  - Do (or don't) represent the game in extensive form
  - How many strategies for player i?
  - Do (or don't) represent the game in normal form
  - What are the SPNE of the repeated game?
  - How does the presence of the B strategy influence the SPNE?

Roadmap

Stage game

Stage game

- set I of players, $i \in I$,
- set A^i of actions for player i,
- $A = \prod_i A^i$, $A^{-i} = \prod_{j \neq i} A^j$,
- payoff function for player i: $g^i : A \to \mathbb{R}$,
- vector payoff function g, $g(a) = (g^i(a))_i$,

Mixed strategies

- $S^i = \Delta(A^i)$, $S = \prod_i S^i$, $S^{-i} = \prod_{j \neq i} S^j$,
- extended payoff function: $g^i : S \to \mathbb{R}$, $g(s) = (g^i(s))_i$,

$$g^i(s) = \mathbf{E}_s g^i(a)$$

- Nash equilibrium: $\tilde{s} \in S$ such that for every i, $s^i \in S^i$

$$g^i(s^i, \tilde{s}^{-i}) \leq g^i(\tilde{s})$$

Information, histories, pure strategies

Histories

- The set of histories of length t is $H_t = A^t$.
- By convention, $H_0 = \{\emptyset\}$ (or $\{*\}$, or anything you want).
- An element $h_t = (a_1, \ldots, a_t) \in H_t$ describes the information available to all the players prior to playing in stage $t + 1$.

Pure strategies

- Player i's behavior at stage t is described by a map $f_t^i \colon H_{t-1} \to A^i$
- A pure strategy for i in the (infinitely) repeated game is $f^i = (f_t^i)_{t \geq 1}$
- Equivalently, it is a mapping $f^i \colon \cup_{t \geq 0} H_t \to A^i$
- Let Σ^i be the set of pure strategies for player i

The "right" notion of strategies

What is a randomized strategy ?

- A mixed strategy for player i is a randomization over pure strategies, element of $\Delta(\Sigma^i)$.
- A behavioral strategy is a mapping from $\cup_{t\geq 0} H_t$ to $\Delta(A^i)$ describing the randomization at each stage.
- More generally, a randomized strategy is a randomization over behavioral strategies.
- So what is the "right" notion?

Kuhn's Theorem

In a game with perfect recall, for every mixed strategy of player i, there exists an equivalent behavioral strategy of player i, and vice versa.Equivalent means, no matter the behavior of other players, the probability distributions induced over histories are the same.

Repeated games without perfect recall are also interesting! (bounded rationality), but beyond the scope of this course.

Roadmap

Induced histories, preferences

Histories

A profile f of p.s. induces a play $h_\infty = (a_1, \ldots, a_t, \ldots)$ in the r.g.

- $a_1 = f(\emptyset) = (f^i(\emptyset))_i$, $h_1 = (a_1)$,
- $a_2 = f(h_1)$, $h_2 = (a_1, a_2)$
- $a_t = f(h_{t-1})$, $h_t = (a_1, \ldots, a_t)$

Payoffs

- G_n game played n times,

$$\gamma_n = \frac{1}{n} \sum_{t=1}^{n} g(a_t)$$

- E_n set of Nash Equilibrium (vector) payoffs of G_n
- What can we say about E_n. for a fixed value of n, for n large?

Example, Prisoner's Dilemma

	C	D
C	$3, 3$	$-1, 4$
D	$4, -1$	$0, 0$

What are E_1, E_2, E_n?

A first general (negative) result

Defending a payoff

Player i can defend a payoff x^i in G if, for every s^{-i} in S^{-i}, there exists $s^i \in S^i$ such that

$$g^i(s^{-i}, s^i) \geq x^i$$

Anti-Folk Theorem

Assume that G has a unique Nash Equilibrium payoff x, and that for every i, player i can defend x^i, then for every n,

$$E_n = \{x\}.$$

Proof

Let f be a NE of G_n in (wlog, pure) strategies, inducing a sequence of actions $a_1, \ldots, a_n$.We show by induction on k that $\forall k, g(a_{n-k}) = x$.

- true for $k = 0$
- $k - 1 \rightarrow k$, assume a_{n-k} is not a NE of G,find a profitable deviation for a player in G_n.

Subgames and subgame perfection

Subgames and continuation strategies

- To each history h_τ, $\tau \leq n-1$ corresponds a subgame of G_n
- A strategy f^i for player i induces a strategy $f^i_{|h_\tau}$ in the repeated game starting after h_τ given by

$$f^i_{|h_\tau}(h_t) = f^i(h_\tau \cdot h_t)$$

where $h_\tau \cdot h_t$ is the history h_τ followed by h_t.

Subgame perfect Nash equilibria

- f is a SPNE of G_n if, for every history h_τ, $\tau \leq n-1$, the profile of continuation strategies $f_{|h_\tau} = (f^i_{|h_\tau})_i$ is a Nash equilibrium of $G_{n-\tau}$.
- E'_n denotes the set of SPNE payoffs of G_n.

Back to an example and a general result

	H	U
G	$1,1$	$-1,-1$
S	$2,0$	$-1,-1$

The game G_2 has a Nash equilibrium which is not a SPNE. In all SPNE of G_2, players play (S,H) at every stage, and after every history.
Can we generalize the result to G_n?
Compare with the SPNE of the two stage repetition of

	H	U	B
G	$1,1$	$-1,-1$	$-5,-5$
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B	$-5,-5$	$-5,-5$	$-4,-4$

Anti-Folk part deux

Assume that G has a unique Nash equilibrium, $\tilde{s}$. Then, G_n has a unique SPNE, and at this SPNE, $\tilde{s}$ is played after every history.

Conclusion

- Repetition can lead to cooperation in a repeated game,
- "end of game" effect can lead to unraveling and prevent cooperation
- Repetition does not necessarily lead to cooperation
- How about infinite (never-ending) games?